

# Large-Scale Simulation of a Self-organizing Neural Network: Formation of a Somatotopic Map

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## Abstract:

The "somatotopic map" of the body surface of animals and humans reflects an ordered, neighborhood preserving connectivity between tactile skin receptors and cortical neurons. The border between adjacent areas representing different body parts changes even in adult animals, which shows, that the map is dynamically maintained, and makes this system an interesting model for the investigation of cortical plasticity.

In this paper we present a large-scale simulation study of a neural network model for the formation and readaptation of a somatotopic map of the inner hand surface. The neural network, which is based on an algorithm developed by Kohonen [3,4], contains 16,384 neurons and 800 tactile receptors, giving a total of 13,107,200 adaptive connections.

The simulation assumes random initial connections between the receptors and neurons, and the formation of the map proceeds during randomly applied, local stimuli. The number of necessary adaptive steps was surprisingly small, which indicates, that a high dimensional input space facilitates the ordering process. Neurons with double and multiple receptive fields can emerge during simulation. The model network readapts upon partial deprivation of sensory input much in the same way as is found in experiments [8].

## 1. Introduction

Topographic representations of sensory surfaces within the cortex are a widespread architectural feature of the brain of higher animals. They can be found in nearly all sensory and motor areas within the brain, e.g. the visual, auditive and somatosensory fields as well as in the motor-cortex [1,2,12,13,14], and it is an intriguing notion that maps of more abstract features might play a role on higher processing levels.

In the somatosensory system the complete body surface is mapped onto a certain part of the cortex called somatosensory cortex. In a series of experiments on the cortical representation of the hand surface in owl monkeys Merzenich et al. [7,8] characterized this topographic representation and showed, that the somatotopic map is dynamically maintained even in adult animals. After depriving a certain area of the cortex from its sensory input (e.g. by nerve section or by amputation of a digit) part of this area is first "silent" upon stimulation of the receptor surface, but within the following months the representations of the adjacent digital areas expand and approach each other in the reorganizing map.

Several models have been designed to explain the formation and plasticity of topographic maps within the cortex. Malsburg and Willshaw [5] introduced a model to explain the development of an (anatomical) topographical projection by fibers growing out of a neuronal sheet into another layer. Pearson et al. [9] proposed a medium scale neural network model of locally connected excitatory and inhibitory cells receiving projections from two receptor sheets corresponding to the glabrous and dorsal surfaces of the hand. A certain topographic order was established at the beginning but the receptive fields were still large and the projection fuzzy. Following repeated stimulation of the receptor sheet the "model cortex" breaks into a pattern of "neuronal groups" of specific cells with similar receptive fields forming a topographic map. Contrary to the Malsburg-Willshaw model part of the formation process was due to an input-selection mechanism based on synaptic plasticity. A third type of network is based on an algorithm proposed by Kohonen [3,4]. In this algorithm each neuron can access all inputs at the beginning, which corresponds to anatomical connections between all cells and all receptors on the receptor surface. Formation of a map proceeds by input-selection alone. Although this algorithm in its original version does not include much biological detail it embodies a

(minimal) set of clearcut principles, which allow to mimic the experimentally observed behaviour.

In previous work Kohonen's algorithm has been successfully applied to modelling the formation and readaptation of maps in the auditive [6] and somatosensory [10] cortex as well as the formation of maps of more abstract "semantic" features pertaining to language [11]. The simulations were restricted to relatively small networks with maximally a few hundred neurons, which received their input from a low dimensional input space. Although the results showed good qualitative agreement with the experimental findings it remained unclear, how the features of the model system will change, when the system is scaled up to a biologically more realistic size.

In this paper we report a simulation of a fairly large scale neural network comprising 16,384 neurons, which are connected to 800 receptors providing an 800-dimensional input vector. The simulation was carried out on a Connection Machine CM-2 using one processor for each neuron and its connections.

## 2. Model and Algorithm:

Our model consists of a hand-shaped sensory surface containing 800 randomly distributed tactile receptors and a two-dimensional grid of 16,384 formal "neurons" ( $128 \times 128$ ) modelling the corresponding somatosensory area within the cortex. Each neuron is connected to each receptor leading to a total of 13 million adaptive connections, whose initial strengths are chosen at random and which are modified during the simulation. The tactile stimuli are modelled by localized Gaussian excitation profiles, which describe the output  $r_i$  of receptor  $i$  at position  $\vec{x}_i$  as a function of its distance from the center  $\vec{x}_s$  of the stimulus by:

$$r_i = A \exp [-(\vec{x}_i - \vec{x}_s)^2 / \sigma_r^2] \quad (1)$$

The width  $\sigma_r$  and the "intensity"  $A$  of the stimuli were held constant throughout the simulation. For each adaptation step the center  $\vec{x}_s$  of the stimulus was chosen at random. Each neuron  $(k, l)$  at position  $\vec{y}_{kl}$  of the neuronal sheet computes a weighted sum:

$$o_{kl} = \sum_i w_{kli} r_i \quad (2)$$

over all receptor outputs, where  $w_{kli}$  denotes the connection strength from receptor  $i$  to neuron  $(k, l)$ . The input for each neuron is therefore described by an 800-dimensional vector  $\vec{r} = (r_1, r_2, \dots, r_{800})^T$ . Following the algorithm of Kohonen the neuron  $(r, s)$  with the maximal sum  $o_{rs}$  is selected and the output  $o_{kl}$  of the neuron  $(k, l)$  is replaced by a Gaussian output function  $h_{rs;kl}$  centered at the position  $\vec{y}_{rs}$  of the selected neuron:

$$h_{rs;kl}(t) = \exp [-(\vec{y}_{rs} - \vec{y}_{kl})^2 / \sigma_h^2(t)] \quad (3)$$

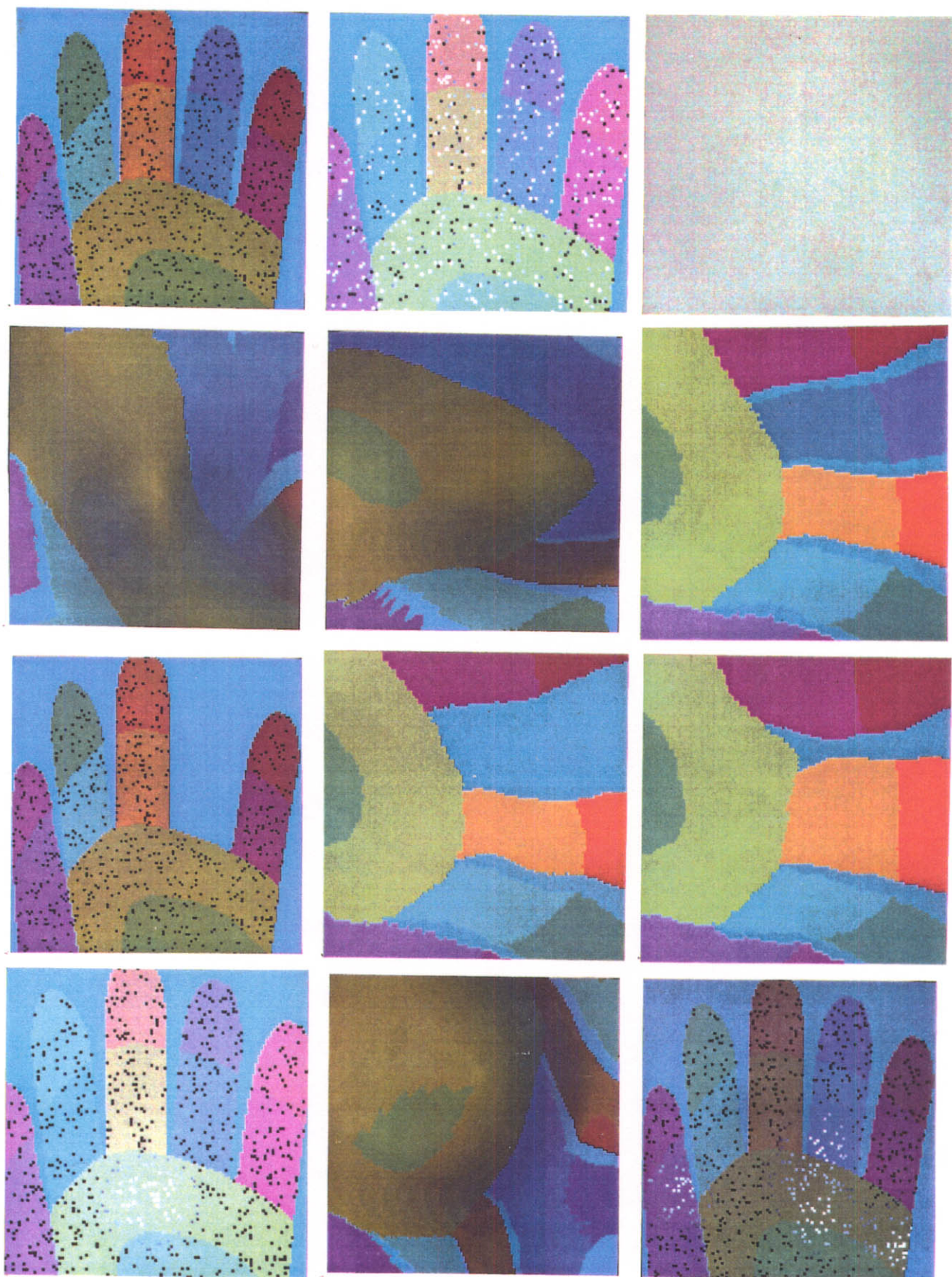
The width  $\sigma_h(t)$  decreases during the simulation from an initial value  $\sigma_i$  to a final value  $\sigma_f$  to allow the neurons to specialize for a certain part of the input space. The introduction of the output-function  $h_{rs;kl}$  is an algorithmic "shortcut" to account for the effect of lateral connections between the neurons []. The connection strengths are then changed according to a Hebb-type learning rule:

$$w_{kli}(t+1) = (w_{kli}(t) + \epsilon(t) h_{rs;kl}(t) \cdot r_i) / \sum_i w_{kli}(t) \quad (4)$$

The learning step width  $\epsilon(t)$  decreases exponentially from an initial value  $\epsilon_i$  to a final value  $\epsilon_f$ .

## 3. Results of the Simulations:

Fig. 4.1 shows the initial state of the network. On the left we see the receptor surface. Each receptor location is indicated by a black dot. In the centermost picture, different brightness values for these receptor dots have been used to indicate the initial random connection strengths from the receptors to a typical neuron. The rightmost picture shows the view on the model cortex. Each pixel represents a neuron  $(k, l)$ , and its gray value coincides with the gray value of the location  $\vec{s}_{kl}$  (defined by:  $\vec{s}_{kl} = \sum_i w_{kli} \vec{x}_i$ ) of its



**Fig.4.1** (top row): Initial state. Left: receptor surface, center: typical receptive field, right: neuronal sheet.  
**Fig.4.2** (row 2): somatotopic map after 200, 1200 and 10000 steps. **Fig.4.3** (row 3): "Amputated hand",  
 and map immediately and 10000 steps after "amputation". **Fig.4.4** (bottom row): left: typical receptive  
 field of adapted map, center: imperfectly ordered map, right: example of multiple receptive field.

receptive field center in Fig. 4.1 (left). The almost uniform gray results from the initial connectivity: each cell's receptive field is diffusely extended over the entire receptor surface. Consequently all positions  $\vec{s}_{ki}$  gather in the center of the sensory surface.

After about 200 stimuli the neurons begin to specialize (Fig. 4.2 left). The receptive fields shrink as the cells get more and more specific, but cells with double and multiple receptive fields may develop, especially during a stage, where the map is fragmented and several representations of the sensory surface compete (see below). After 1200 stimuli a crude map of the hand-surface has emerged with four fingers already present in the correct topographic order (Fig. 4.2 center), but it took about 10,000 stimuli to complete and stabilize the map (Fig. 4.2 right). Parameter values of this simulation were  $\sigma_i=40$ ,  $\sigma_f=20$  (units of the  $128 \times 128$  grid),  $\sigma_r=0.12$ ,  $\epsilon_i=0.2$ ,  $\epsilon_f=0.1$ .

The number of adaptive steps necessary to form the map is less than the number of cells in the network. Previous simulations of Kohonen networks using low-dimensional input vectors required a much larger number of steps (20,000 steps for a network with 900 cells and a two-dimensional input-space [10]) although the network was much smaller. Note that  $\sigma_h$  is only 30% of the diameter of the receptor surface. These results indicate, that a high-dimensional input space strongly facilitates the topographic ordering process.

Fig. 4.3 shows the simulation of the "amputation" experiment performed by Merzenich et al. [8] ( $\sigma_i=20$ ,  $\sigma_f=10$ ,  $\sigma_r=0.12$ ,  $\epsilon_i=0.1$ ,  $\epsilon_f=0.05$ ). The fourth digit of our model hand was removed (Fig. 4.1 left), depriving the neurons of the corresponding part of the neuronal sheet from their sensory input (Fig. 4.3 center). During subsequent stimulation the somatotopic map gradually readapts and the expanding representations of the adjacent areas approach each other. After 10,000 further stimuli the new map has stabilized (Fig. 4.3 right). It is interesting to note, that the "cortical" reorganization almost exclusively affects the neighboring areas of the deprived region, although each cell is in principle connected to each receptor: The size of the "digit 1" and "digit 2" representations changes little, while the size and location of the "digit 3" and "digit 5" areas changes dramatically as it is found in the corresponding experiments.

Fig. 4.4 (left) shows a typical receptive field for a neuron in the topographic map of Fig. 4.2 right. It is well localized, with width roughly of the diameter of the applied stimulus. Note that although most of the synaptic strengths decay to zero except for those whose receptors form the receptive field, each cell keeps its connections to all receptors so that synapses can be "revived" if the stimulus distribution changes.

Fig. 4.4 (center) shows the result of another simulation leading to an incompletely ordered map. The sensory surface is mapped twice to the neuronal sheet: two representations of the second, third and fourth digit are visible. Configurations like this are generally accompanied by patches of cells with double or multiple receptive fields (Fig. 4.4 right). Cells of this type allow to map distant locations on the receptor surface onto neighboring locations of the cortical sheet without introducing a discontinuity in the response properties of the neurons and have been found within the somatosensory cortex, but they are seldom, perhaps because of their correlation with defects in the topographic maps.

#### 4. References:

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